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Some quadrature-based versions of the generalized Newton method for solving nonsmooth equations

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ABSTRACT

In this paper, modifications of a generalized Newton method based on some rules of quadrature are studied. The methods considered are Newton-like iterative schemes for numerical solving systems of nonsmooth equations. Some mild conditions are given that ensure superlinear convergence to a solution. Moreover, a parameterized version of the midpoint version is presented. Finally, results of numerical tests are established.

1. Introduction

We consider some iterative methods for solving the system of n nonlinear equations with n variables

$$F(x) = 0, \tag{1}$$

where $F : R^n \to R^n$ is assumed to be Lipschitz continuous. Throughout the whole paper we assume that there exists $x^* \in R^n$ such that $F(x^*) = 0$.

One of the best-known methods for solving (1) is the generalized Jacobian-based Newton method. Such superlinearly convergent methods for solving semismooth systems were proposed in [1–4]. The fundamental form of the method is defined by

$$x^{(k+1)} = x^{(k)} - (V^{(k)})^{-1}F(x^{(k)}), \quad k = 0, 1, \ldots \tag{2}$$

where $V^{(k)}$ could be taken as an element of some subdifferential of F at $x^{(k)}$. It was assumed to be an element of the Clarke generalized Jacobian (Qi and Sun [1]), of the *B*-differential (Qi [2]), of the *b*-differential (Sun and Han [3]) and of the $*$-differential (Gao [4]). Obviously, the increment $x^{(k+1)} - x^{(k)}$ can be obtained as the solution of a linear system with matrix $V^{(k)}$ by any iterative or direct method. Moreover, Xu and Chang in [5], Potra et al. [6], Śmietański in [7] and others introduced practical (i.e. computational) ways of approximating various subdifferentials. In recent years some authors have done interesting research on the nonsmooth equations (cf. [8–10]).

Cordero and Torregrosa in [11] proposed some new variants of Newton's method for solving smooth equations, based on trapezoidal and midpoint rules of quadrature. These methods converge quadratically for a continuously differentiable function F in some neighborhood of the solution x^*. A generalization of methods based on quadrature formulas is presented in [12].

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An approximate Newton method for solving non-smooth equations with infinite max functions

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In this paper, a new version of an approximate Newton method for solving non-smooth equations with infinite max function is presented. This method uses a difference approximation of the generalized Jacobian based on a weak consistently approximated Jacobian. Numerical example is reported for the generalized Newton method using two versions of approximation.

Keywords: non-smooth equation; semismooth function; weak consistently approximated Jacobian; difference approximation; superlinear convergence

2000 AMS Subject Classifications: 65H10, 90C33

ACM Computing Classification System Codes: G.1.5; G.1.2; F.2.1

1. Introduction

Many important practical problems of mathematical programming require the solving of a non-linear system of equations

$$F(x) = 0, \tag{1}$$

where $F : R^n \to R^n$ is locally Lipschitz.

The generalized Jacobian method for solving such systems was proposed by Qi and Sun [13] in the form

$$x^{(k+1)} = x^{(k)} - V_k^{-1} F(x^{(k)}), \quad V_k \in \partial F(x^{(k)}), \tag{2}$$

where $\partial F(x^{(k)})$ is the generalized Jacobian of the function F at $x^{(k)}$, defined by Clarke [5], and a matrix V_k is taken arbitrarily from $\partial F(x^{(k)})$. The iteration generated by Equation (2) is locally superlinearly convergent under the assumption of semismoothness of the function F.

Xu and Chang [16] expanded method (2), replacing an iteration matrix with an adequately defined consistently approximated Jacobian, to avoid the complicated evaluations of V_k. Xu and

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ORIGINAL PAPER

Convergence of a generalized Newton and an inexact generalized Newton algorithms for solving nonlinear equations with nondifferentiable terms

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Abstract In this paper, we consider two versions of the Newton-type method for solving a nonlinear equations with nondifferentiable terms, which uses as iteration matrices, any matrix from B-differential of semismooth terms. Local and global convergence theorems for the generalized Newton and inexact generalized Newton method are proved. Linear convergence of the algorithms is obtained under very mild assumptions. The superlinear convergence holds under some conditions imposed on both terms of equation. Some numerical results indicate that both algorithms works quite well in practice.

Keywords Nonsmooth equations · Nondifferentiable terms · Generalized Newton method · Inexact generalized Newton method · Convergence theorem

Mathematics Subject Classifications (2000) 65H10 · 65N22

1 Introduction

Consider the following system of nonlinear equations

$$H(x) = 0, \ x \in D \subset R^n. \tag{1}$$

In general case, a quasi-Newton method for solving nonlinear equations generates a sequence $\{x^{(k)}\}$ by letting $x^{(k+1)} = x^{(k)} + s^{(k)}$, where a search direction $s^{(k)}$ is a solution of the system of linear equations

$$B_k s^{(k)} = -H\left(x^{(k)}\right),$$

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